A Brief Introduction to Causal Inference

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causalcourse.com

What is causal inference?

Inferring the effects of any treatment/policy/intervention/etc.

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Inferring the effects of any treatment/policy/intervention/etc.

Examples:

- Effect of treatment on a disease
- Effect of climate change policy on emissions
- Effect of social media on mental health
- Many more (effect of X on Y)

Motivating example: Simpson's paradox

Correlation does not imply causation

Then, what does imply causation?

Causation in observational studies

Simpson's paradox: COVID-27

New disease: COVID-27

Treatment T: A (0) and B (1)

Condition C: mild (0) or severe (1)

Outcome Y: alive (0) or dead (1)

Simpson's paradox: mortality rate table

Treatment	Total
A	**16%** (240/1500)
B	19% (105/550)

$\mathbb{E}[Y|T]$

Simpson's paradox: mortality rate table

Condition

Treatment	Mild	Severe	Total
A	15% (210/1400)	30% (30/100)	**16%** (240/1500)
B	**10%** (5/50)	**20%** (100/500)	19% (105/550)
	$\mathbb{E}[Y \mid T, C = 0]$	$\mathbb{E}[Y \mid T, C = 1]$	$\mathbb{E}[Y \mid T]$

Simpson's paradox: mortality rate table

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	$\mathbb{E}[Y \mid T, C = 0]$	$\mathbb{E}[Y \mid T, C = 1]$	$\mathbb{E}[Y \mid T]$

$$\frac{1400}{1500}(0.15) + \frac{100}{1500}(0.30) = 0.16$$

$$\frac{50}{550}(0.10) + \frac{500}{550}(0.20) = 0.19$$

Simpson's paradox: scenario 1 (treatment B)

Condition

Treatment	Mild	Severe	Total
A	15% (210/1400)	30% (30/100)	**16%** (240/1500)
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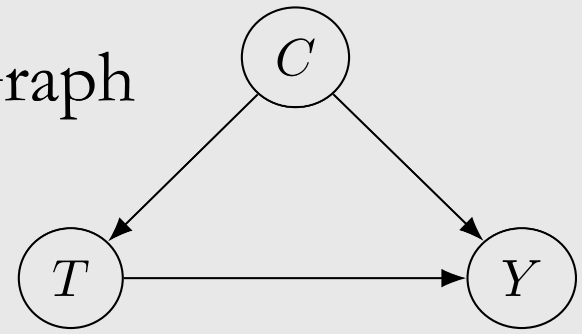
Causal Graph



Simpson's paradox: scenario 1 (treatment B)

Treatment	Condition: Mild	Condition: Severe	Condition: Total
A	15% (210/1400)	30% (30/100)	**16%** (240/1500)
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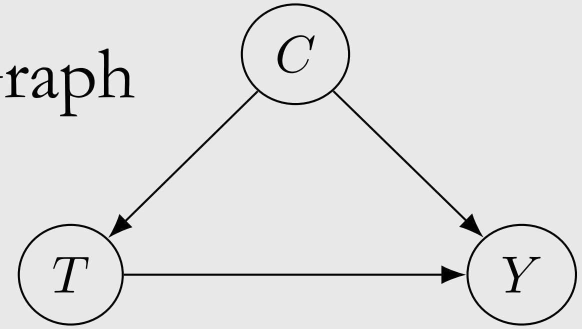

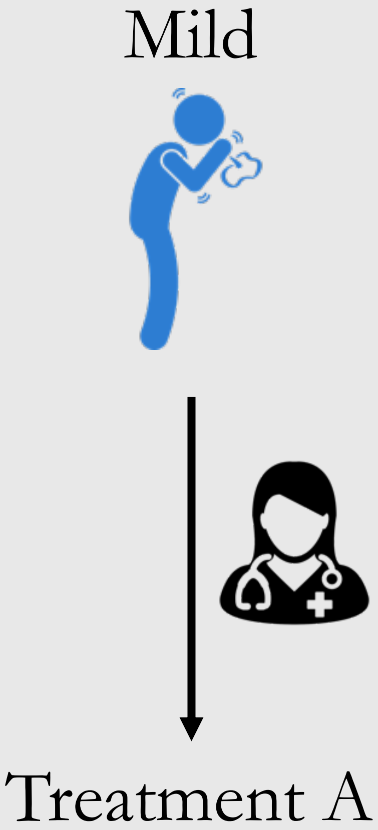

Simpson's paradox: scenario 1 (treatment B)

Treatment		Condition		
		Mild	Severe	Total
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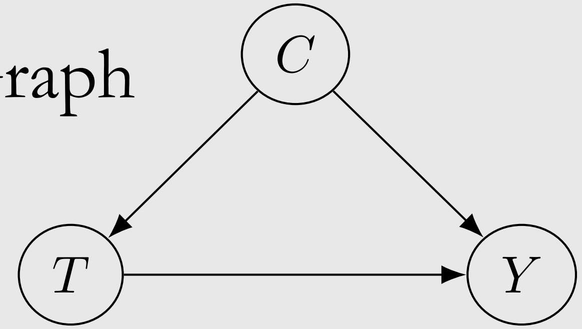

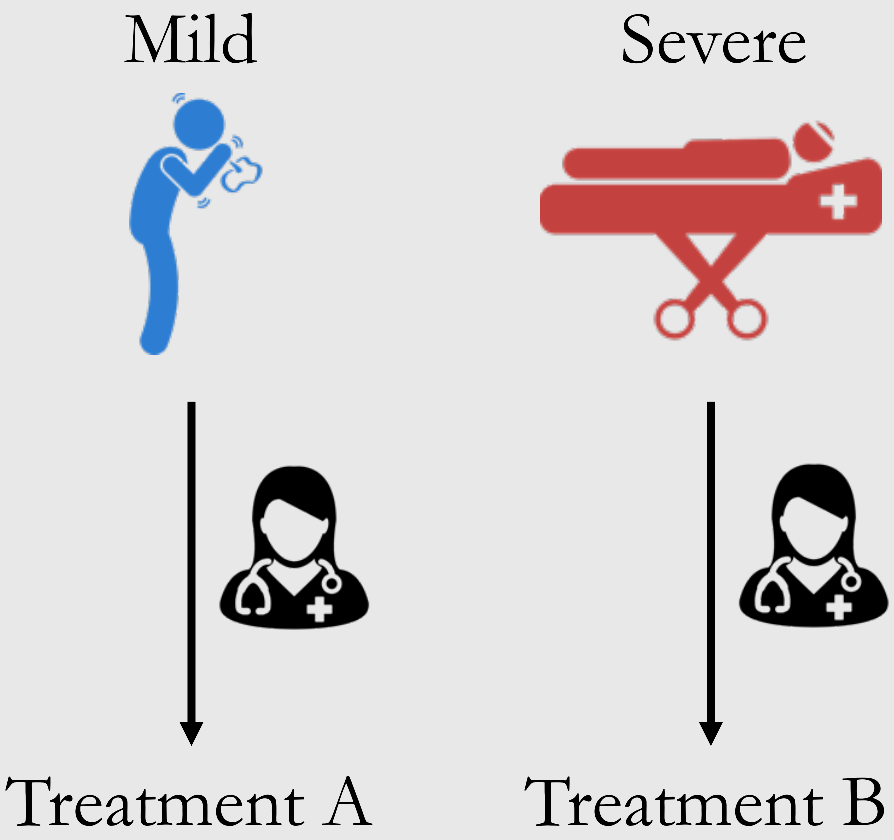

Simpson's paradox: scenario 2 (treatment A)

Condition

Treatment	Mild	Severe	Total
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Simpson's paradox: scenario 2 (treatment A)

Condition

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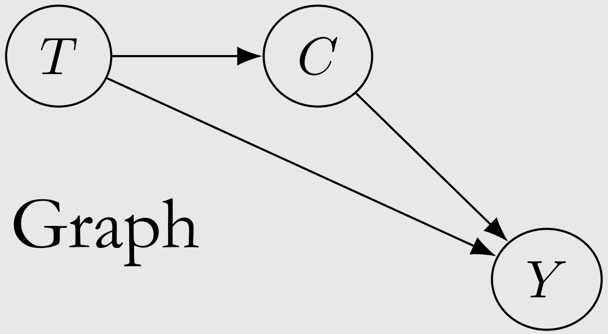

Causal Graph

Simpson's paradox: scenario 2 (treatment A)

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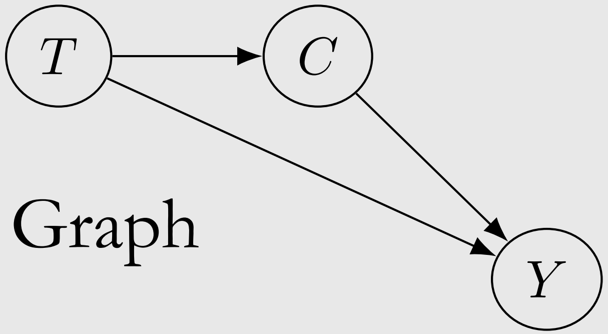

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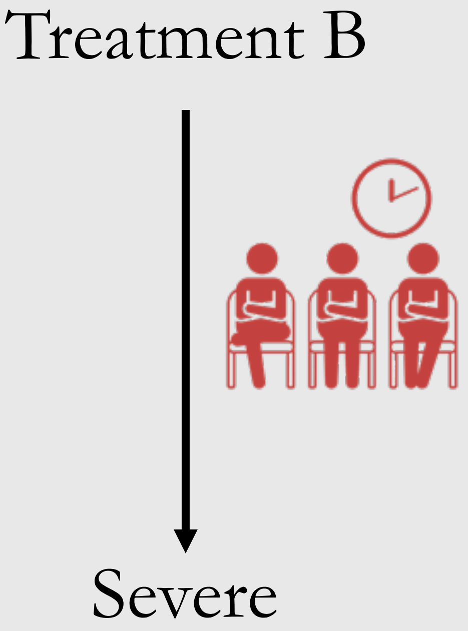

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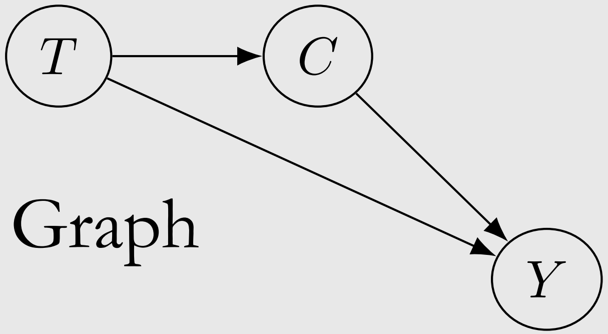

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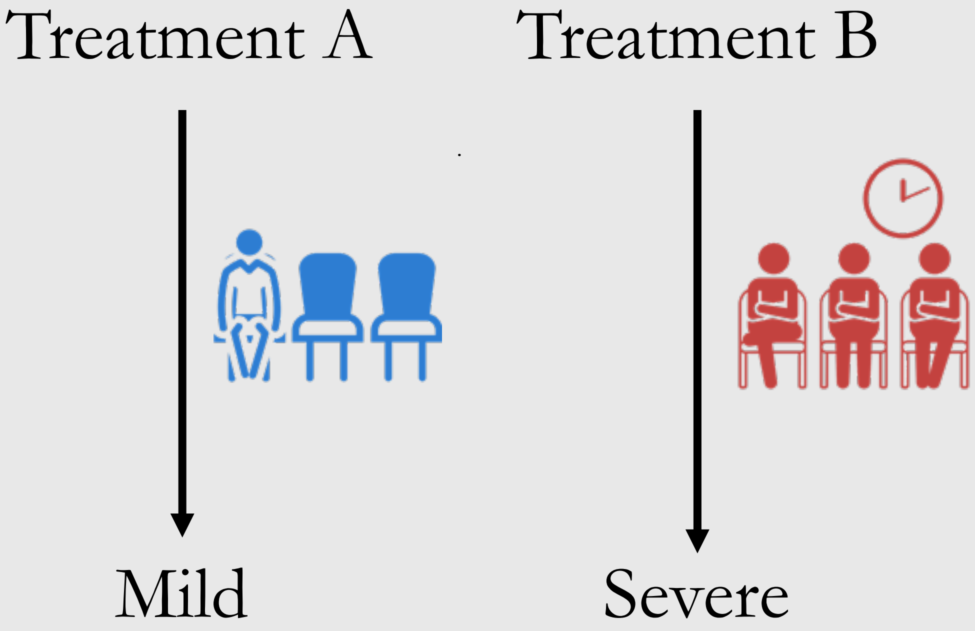

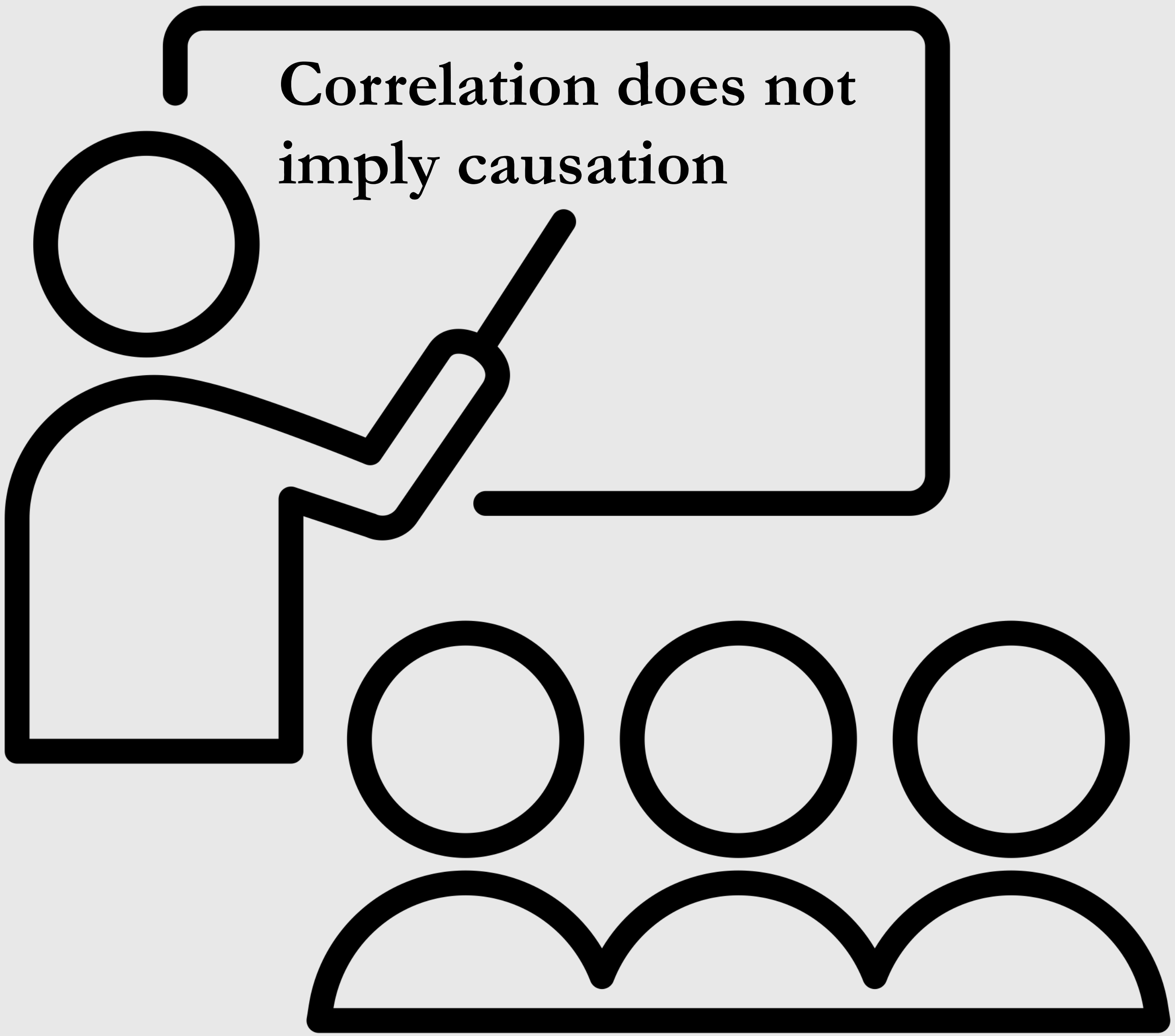
Correlation does not
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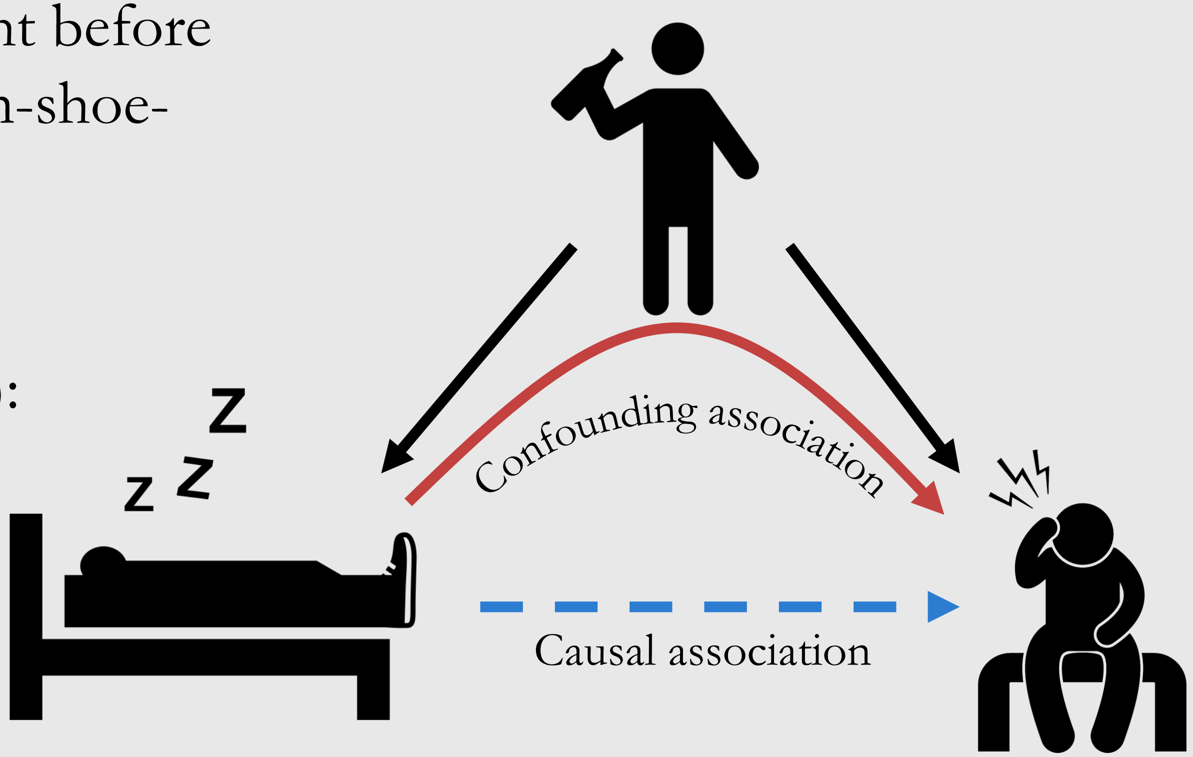
Correlation does not imply causation

Sleeping with shoes on is strongly correlated with waking up with a headache

Common cause: drinking the night before

1. Shoe-sleepers differ from non-shoe-sleepers in a key way
2. Confounding

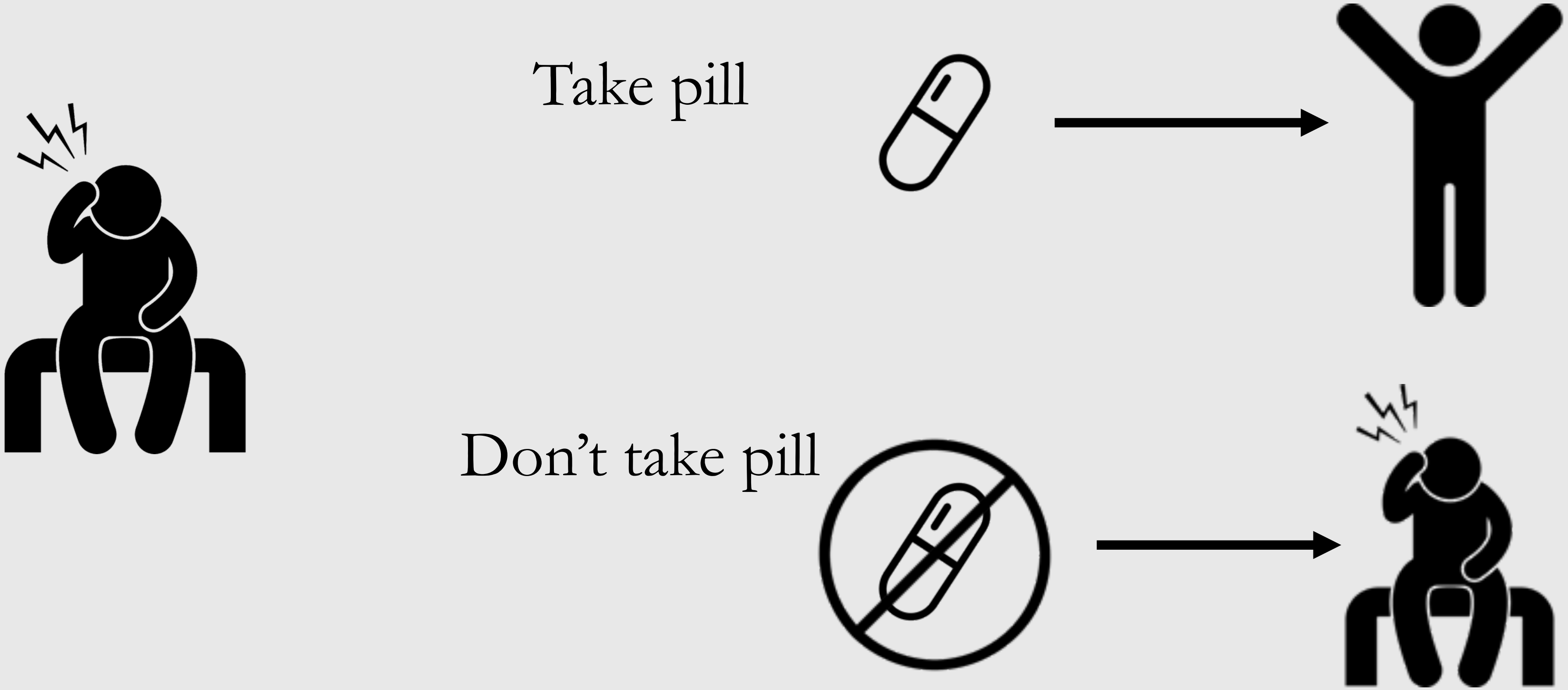
Total association (e.g. correlation): mixture of causal and confounding association



Then, what does imply causation?

Potential outcomes: intuition

Inferring the effect of treatment/policy on some outcome



Potential outcomes: intuition

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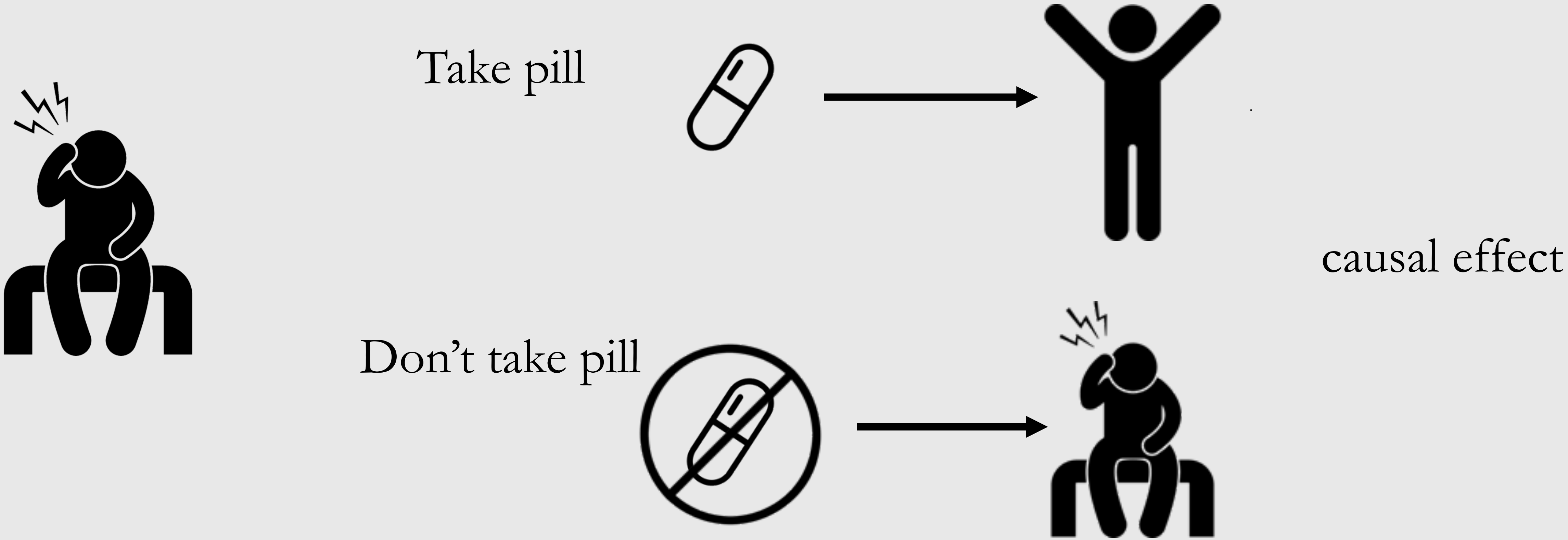
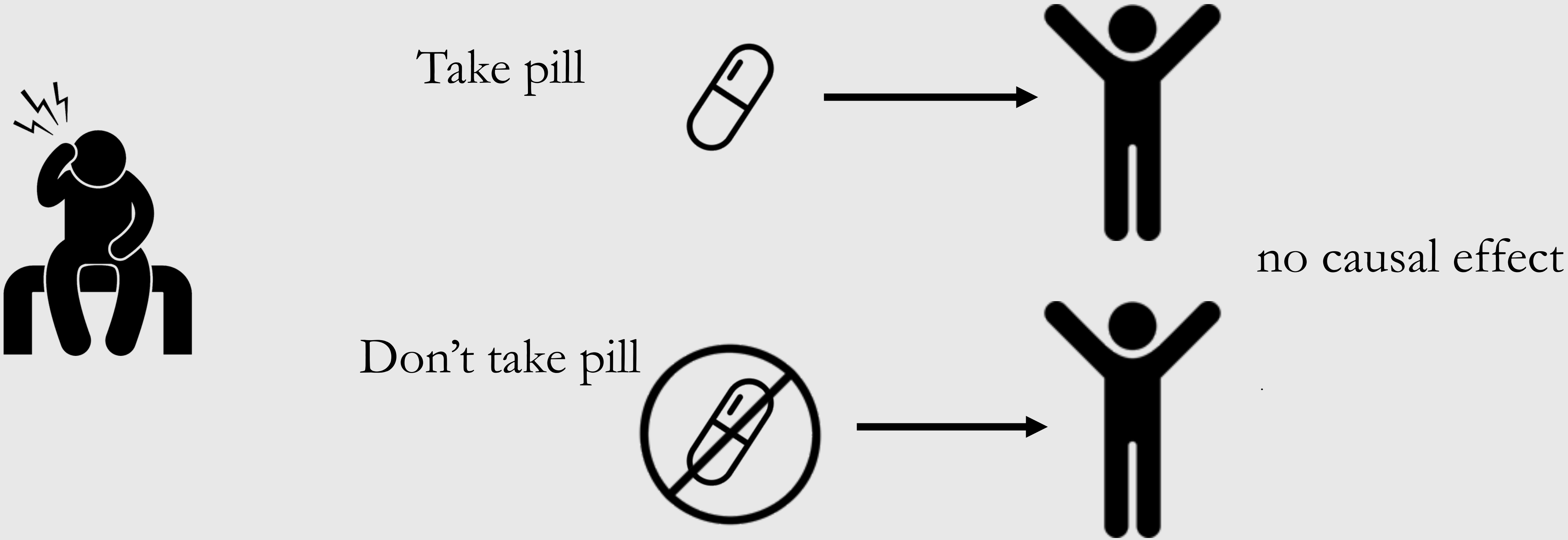
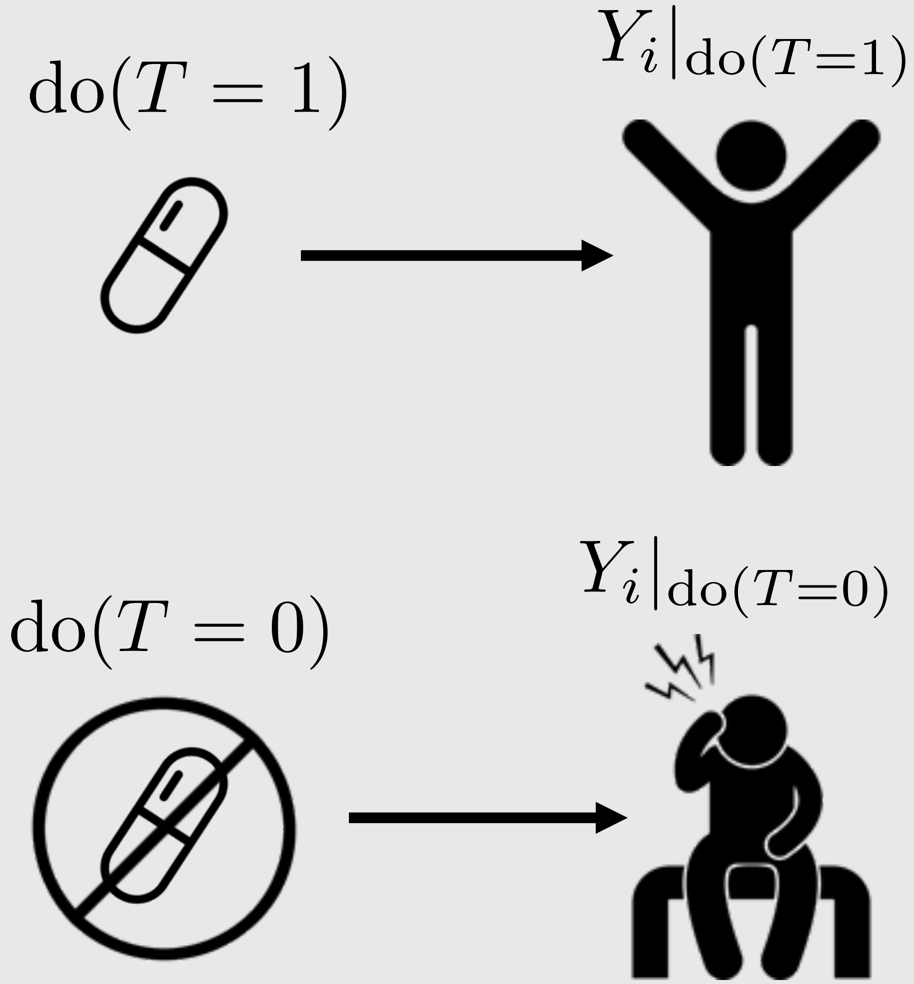


Potential outcomes: intuition

Inferring the effect of treatment/policy on some outcome



Potential outcomes: notation

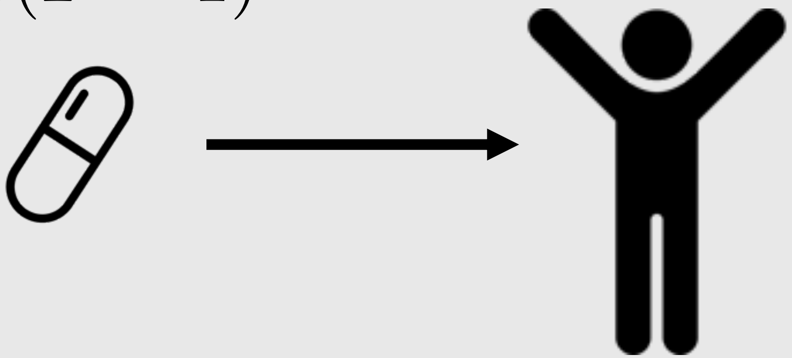


T : observed treatment
Y : observed outcome
i : used in subscript to denote a specific unit/individual

Potential outcomes: notation

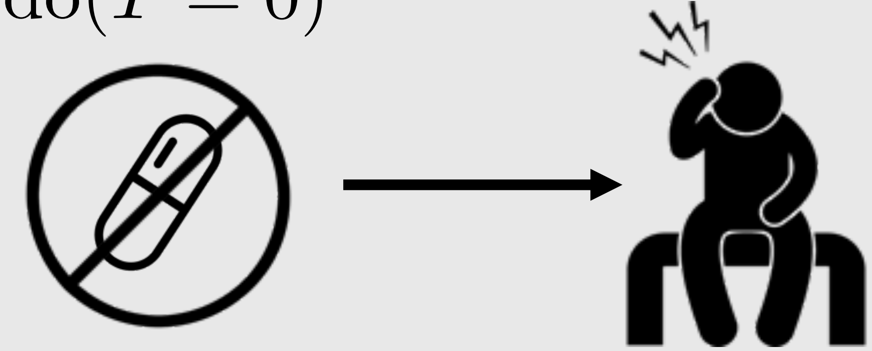
$\text{do}(T = 1)$

$Y_i|_{\text{do}(T=1)} \triangleq Y_i(1)$



$\text{do}(T = 0)$

$Y_i|_{\text{do}(T=0)} \triangleq Y_i(0)$



T	: observed treatment
Y	: observed outcome
i	: used in subscript to denote a specific unit/individual
$Y_i(1)$	: potential outcome under treatment
$Y_i(0)$	: potential outcome under no treatment

Causal effect

$Y_i(1) - Y_i(0)$

Fundamental problem of causal inference

Counterfactual

$\mathrm{do}(T = 1)$ $\quad$ $Y_i(1) = 1$

T	: observed treatment
Y	: observed outcome
i	: used in subscript to denote a specific unit/individual
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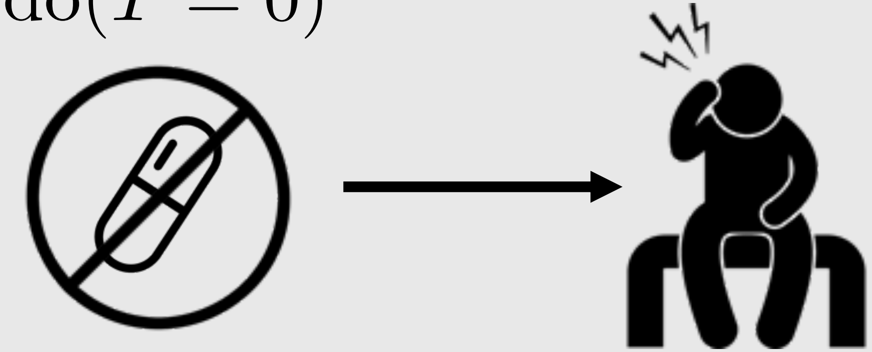
Factual

$\mathrm{do}(T = 0)$ $\quad$ $Y_i(0) = 0$



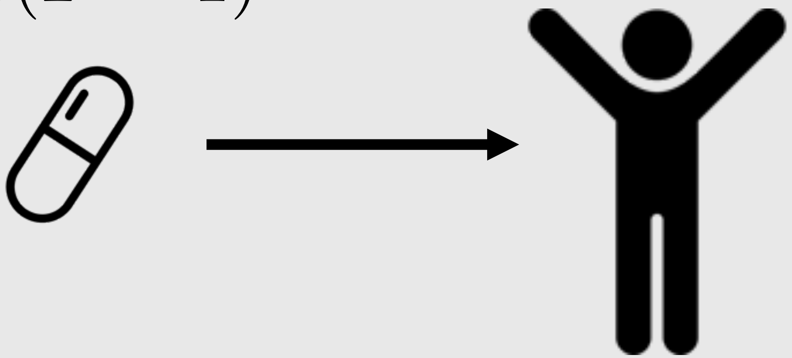
Causal effect

$$Y_i(1) - Y_i(0) = 1$$

Fundamental problem of causal inference

Factual

$\text{do}(T = 1)$ $\quad Y_i(1) = 1$



Counterfactual

$\text{do}(T = 0)$ $\quad Y_i(0) = 0$

T	: observed treatment
Y	: observed outcome
i	: used in subscript to denote a specific unit/individual
$Y_i(1)$	: potential outcome under treatment
$Y_i(0)$	: potential outcome under no treatment

Causal effect

$$Y_i(1) - Y_i(0) = 1$$

Average treatment effect (ATE)

Individual treatment effect (ITE): $Y_i(1) - Y_i(0)$

Average treatment effect (ATE):

$$\mathbb{E}[Y_i(1) - Y_i(0)] = \mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$$
$$\neq \mathbb{E}[Y|T=1] - \mathbb{E}[Y|T=0]$$

T	: observed treatment
Y	: observed outcome
i	: used in subscript to denote a specific unit/individual
$Y_i(1)$	: potential outcome under treatment
$Y_i(0)$	: potential outcome under no treatment
$Y(t)$	: population-level potential outcome

Motivating example: Simpson's paradox

Correlation does not imply causation

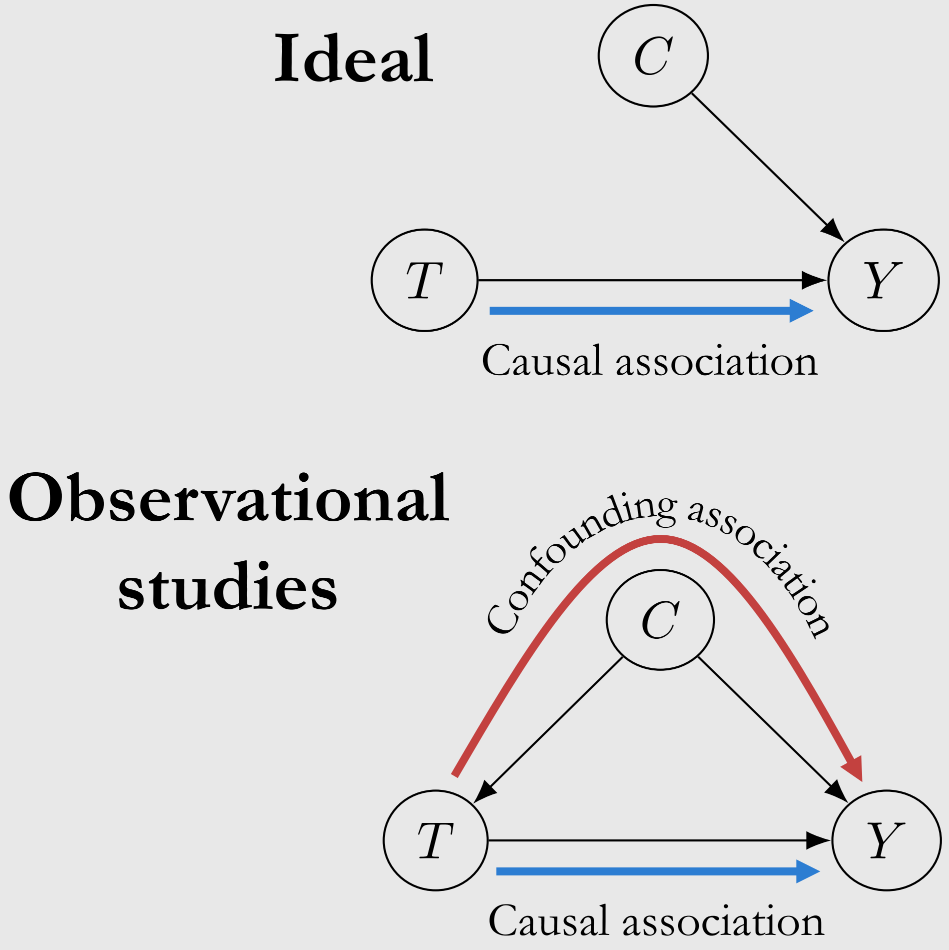
Then, what does imply causation?

Causation in observational studies

Observational studies

Can't always randomize treatment

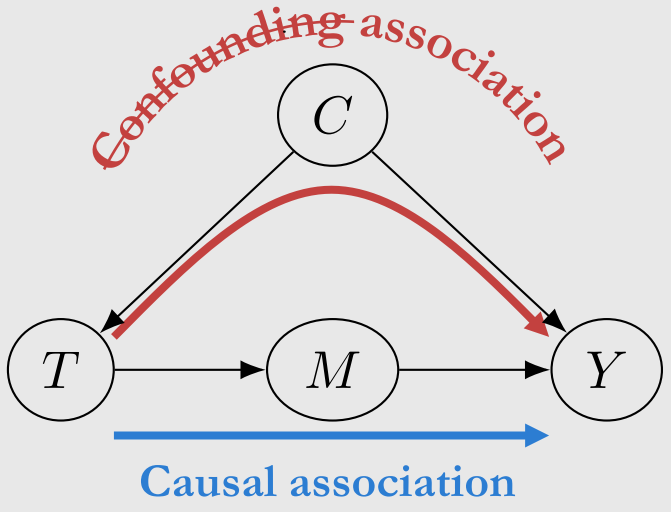
- **Ethical reasons** (e.g. unethical to randomize people to smoke for measuring effect on lung cancer)
- **Infeasibility** (e.g. can't randomize countries into communist/capitalist systems to measure effect on GDP)
- **Impossibility** (e.g. can't change a living person's DNA at birth for measuring effect on breast cancer)



How do we measure causal effects in observational studies?

Solution: adjust/control for confounders

Adjust/control for the right variables W.

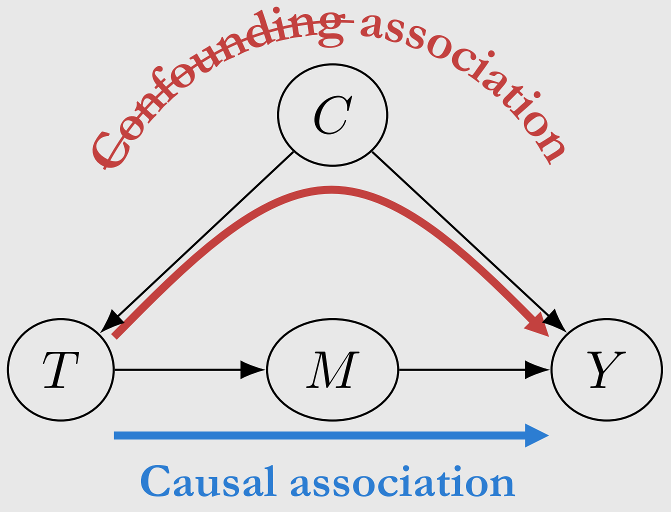


Solution: adjust/control for confounders

Adjust/control for the right variables W.

If W is a sufficient adjustment set, we have

$$\mathbb{E}[Y(t)|W = w] \triangleq \mathbb{E}[Y|\mathrm{do}(T = t), W = w] = \mathbb{E}[Y|t, w]$$



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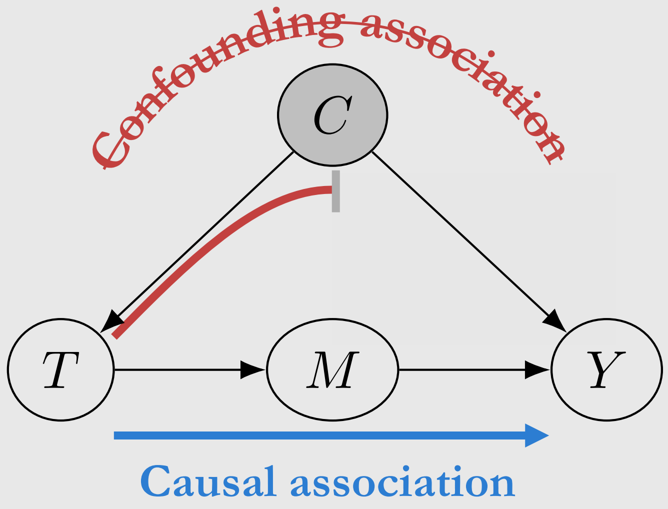

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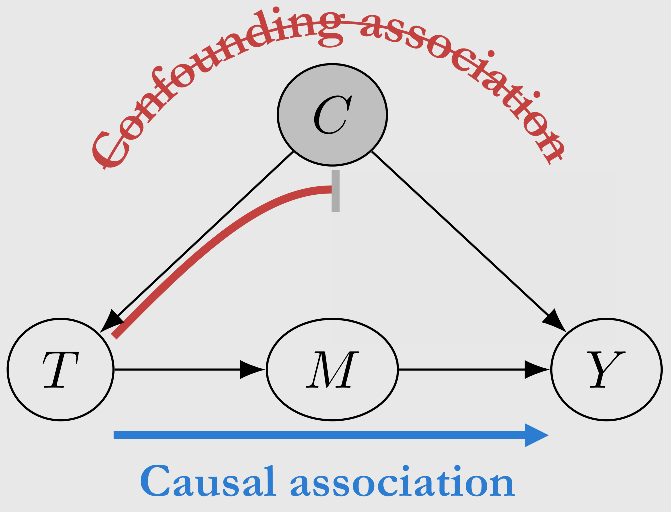
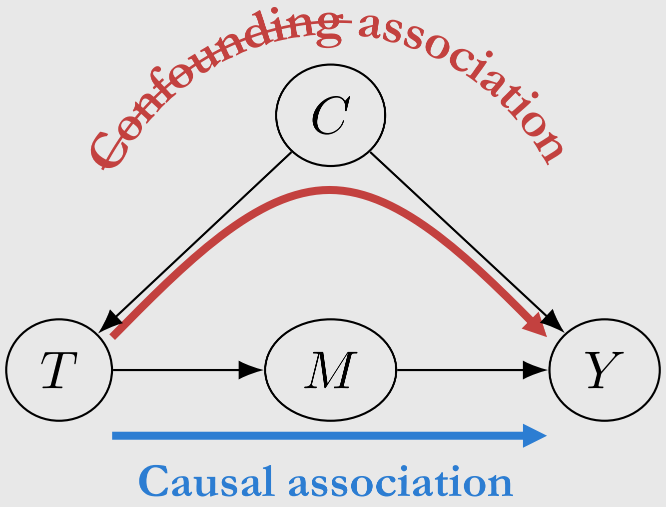
$$\mathbb{E}[Y(t)|W=w] \triangleq \mathbb{E}[Y|\text{do}(T=t), W=w] = \mathbb{E}[Y|t,w]$$

$$\mathbb{E}[Y(t)] \triangleq \mathbb{E}[Y|\text{do}(T=t)] = \mathbb{E}_W \mathbb{E}[Y|t,W]$$



Solution: backdoor adjustment

$$\mathbb{E}[Y|\text{do}(T=t)] = \mathbb{E}_W \mathbb{E}[Y|t,W]$$



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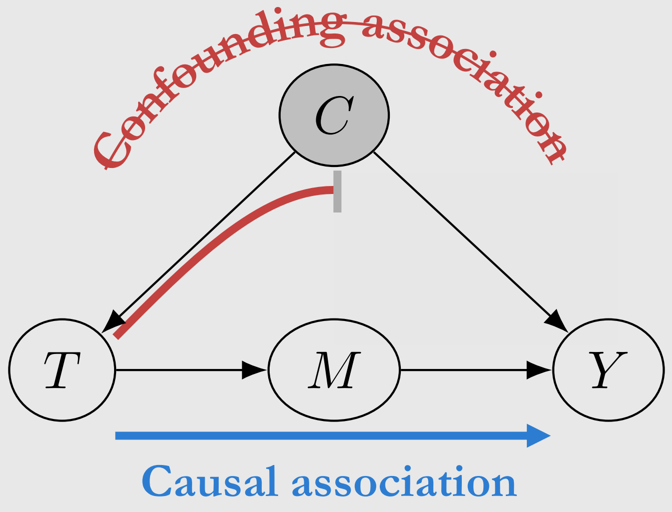

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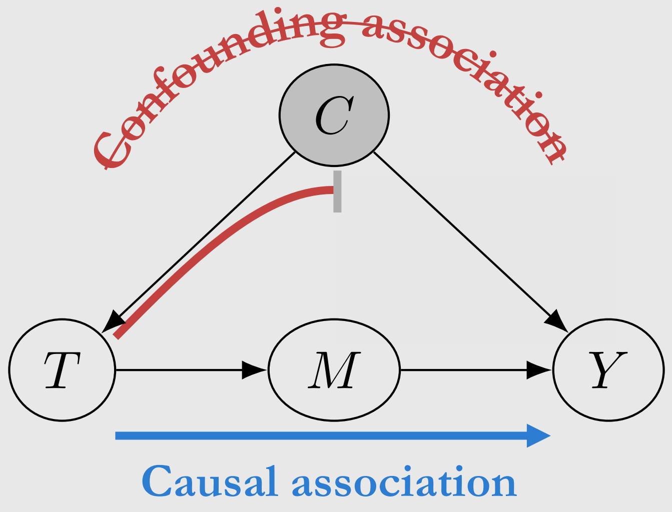
Shaded nodes are examples of sufficient adjustment sets W



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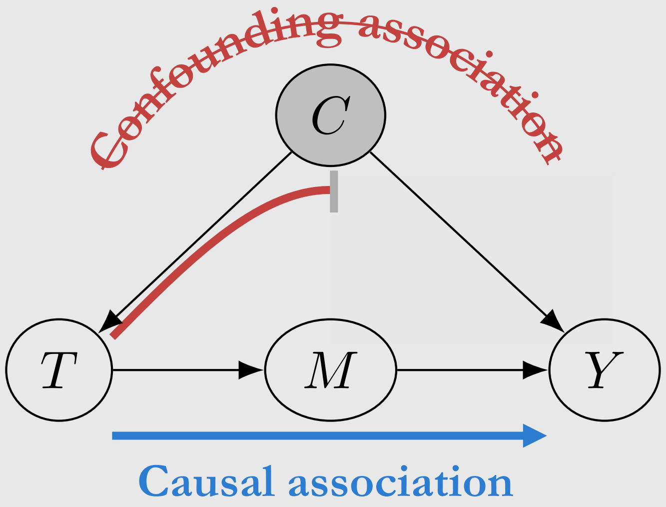

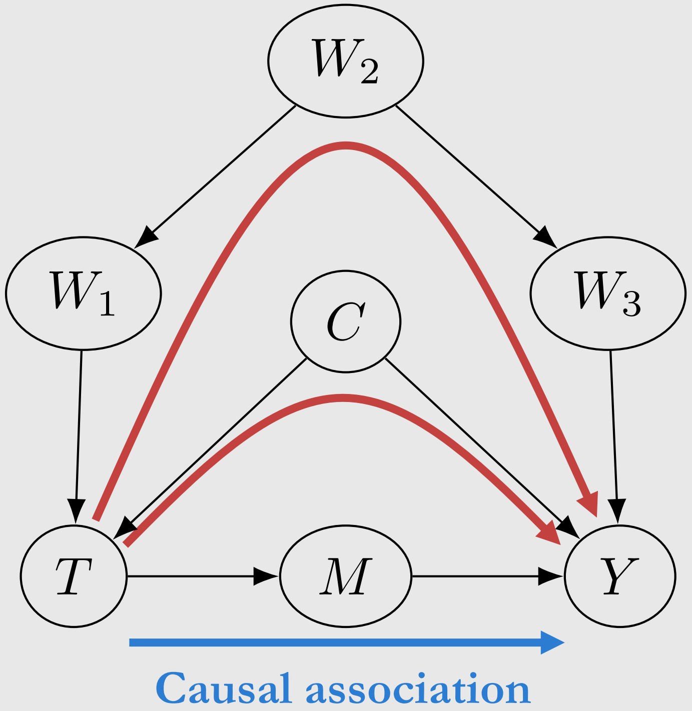

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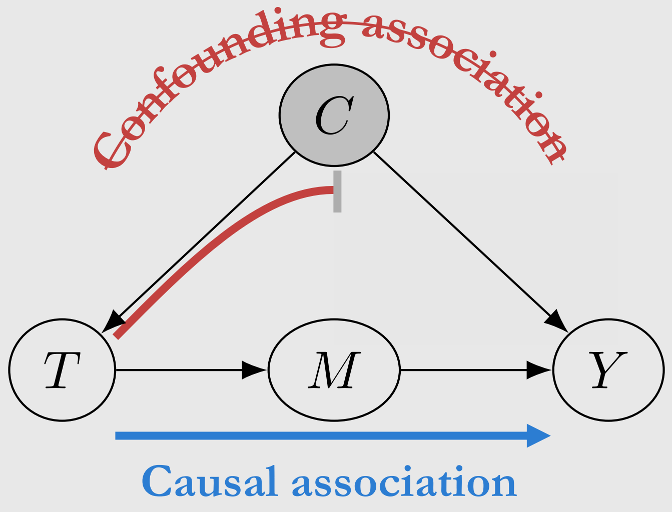

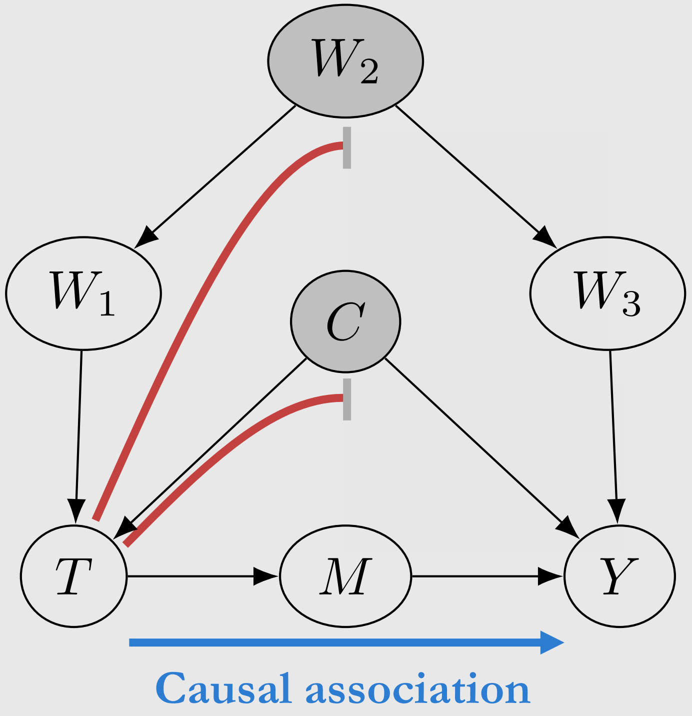

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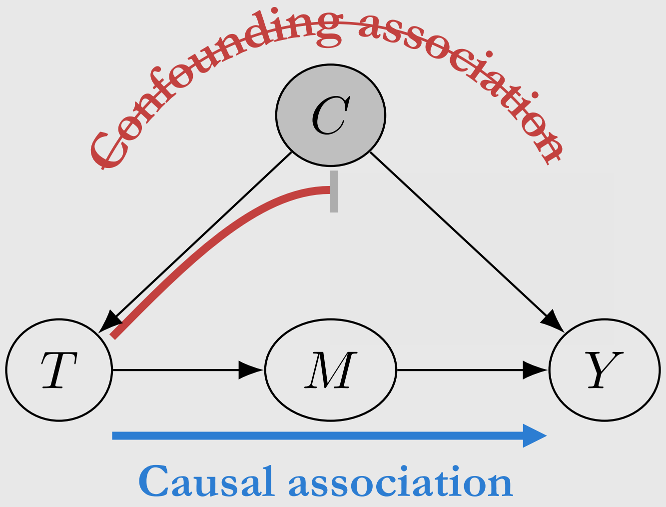

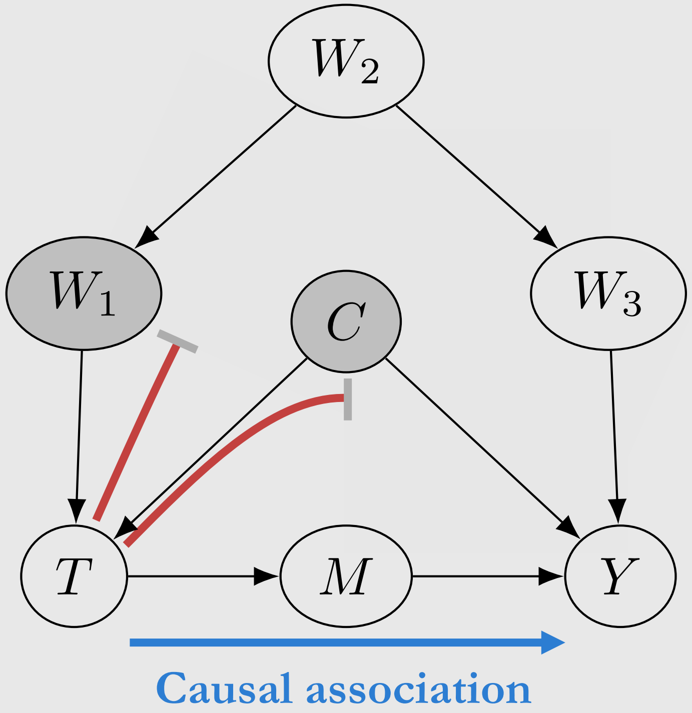

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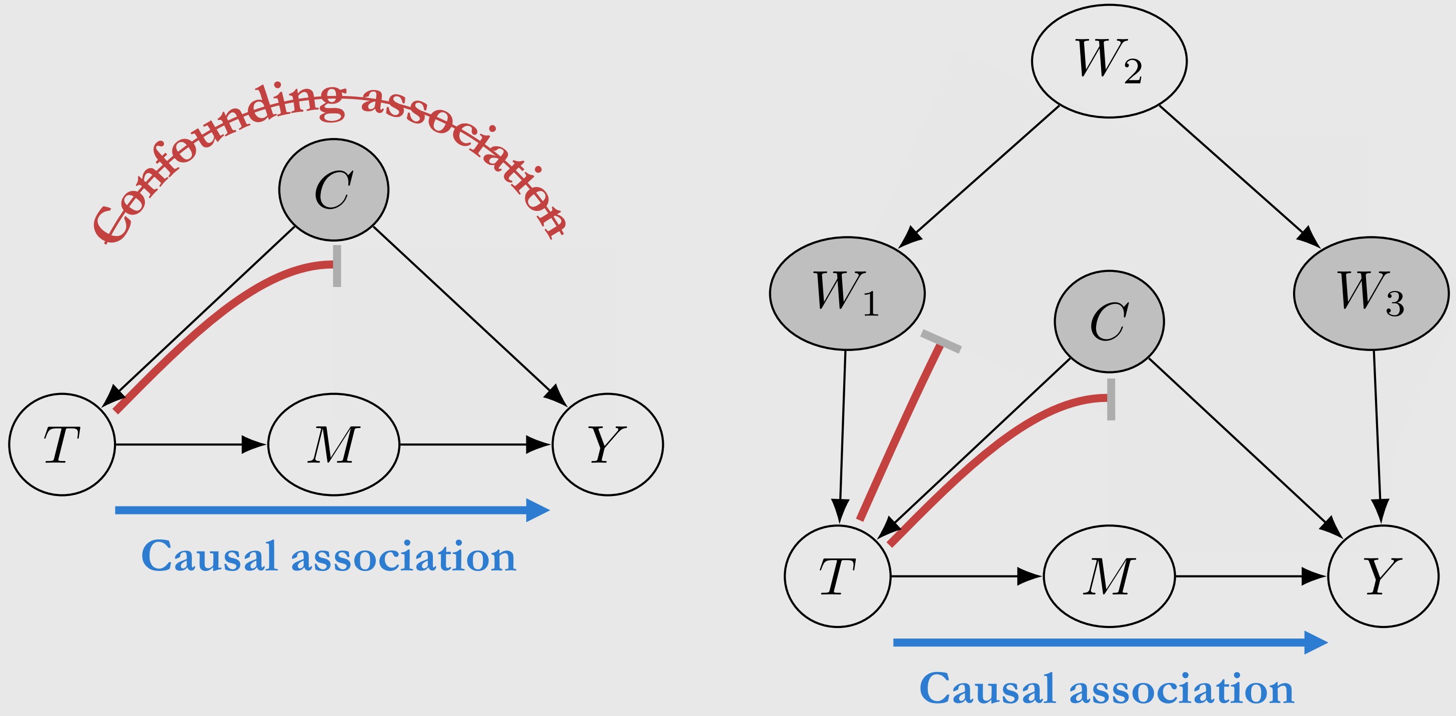

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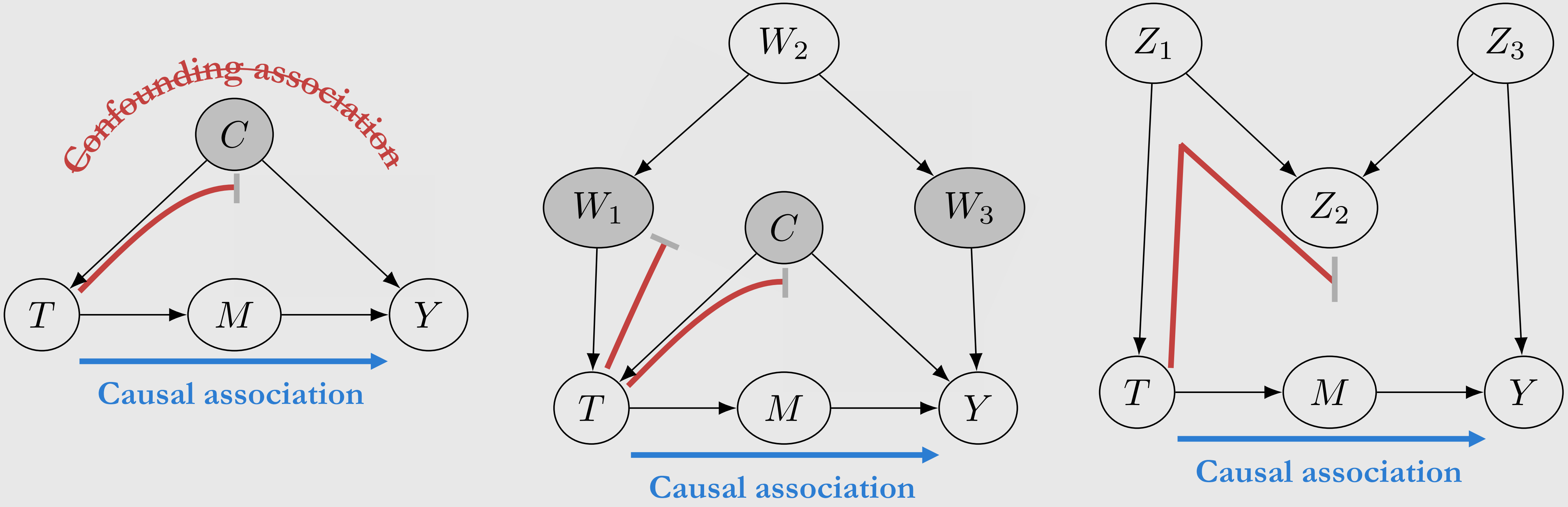
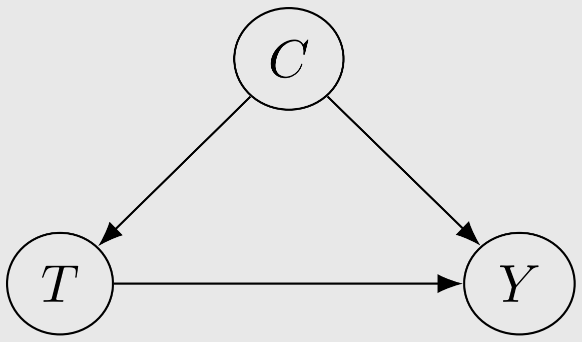


Application to the COVID-27 example

$$\mathbb{E}[Y|\text{do}(T=t)] = \mathbb{E}_C \mathbb{E}[Y|t, C]$$

Causal Graph



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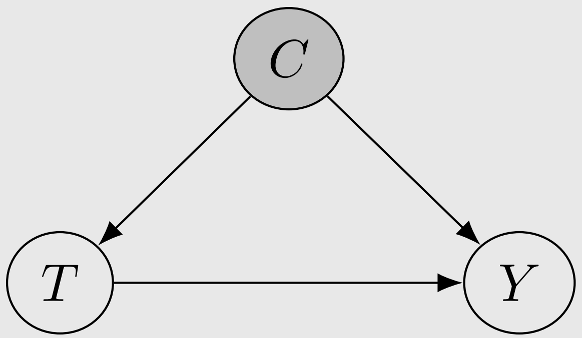

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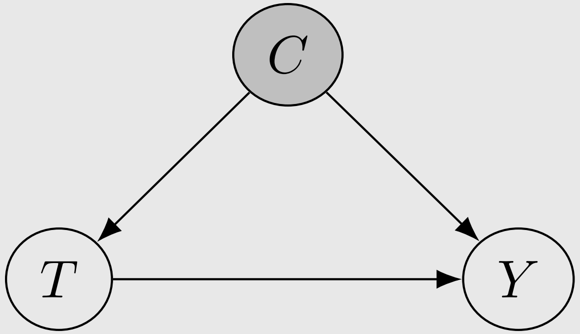
Application to the COVID-27 example

$$\mathbb{E}[Y|\text{do}(T=t)] = \mathbb{E}_C \mathbb{E}[Y|t, C] = \sum_c \mathbb{E}[Y|t, c] P(c)$$

Causal Graph



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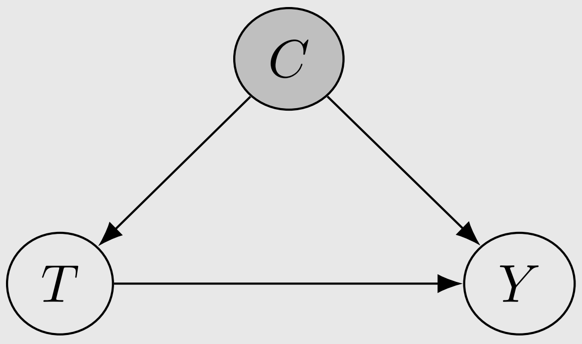

Condition

Treatment	Mild	Severe	Total	Causal
A	15% (210/1400)	30% (30/100)	**16%** (240/1500)	19.4%
B	**10%** (5/50)	**20%** (100/500)	19% (105/550)	**12.9%**
	$\mathbb{E}[Y\|t, C=0]$	$\mathbb{E}[Y\|t, C=1]$	$\mathbb{E}[Y\|t]$	$\mathbb{E}[Y\|\text{do}(t)]$

Application to the COVID-27 example

$$\mathbb{E}[Y|\text{do}(T=t)] = \mathbb{E}_C \mathbb{E}[Y|t,C] = \sum_c \mathbb{E}[Y|t,c]P(c)$$

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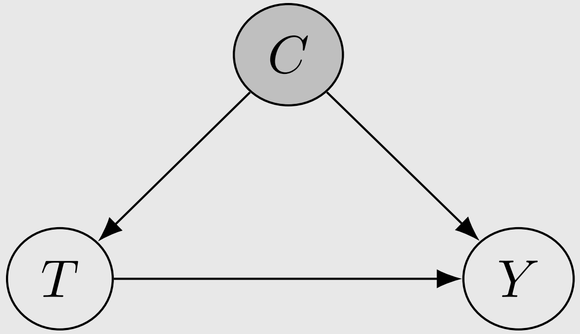

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B	**10%** (5/50)	**20%** (100/500)	19% (105/550)	**12.9%**
	$\mathbb{E}[Y|t,C=0]$	$\mathbb{E}[Y|t,C=1]$	$\mathbb{E}[Y|t]$	$\mathbb{E}[Y|\text{do}(t)]$

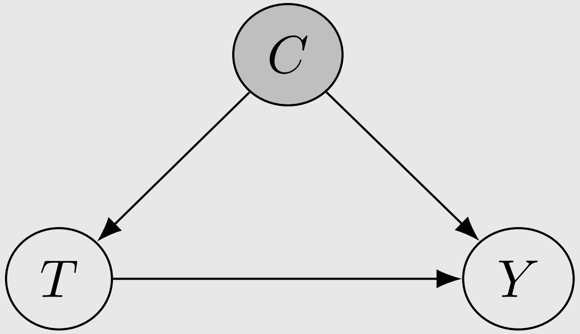
$$\frac{1450}{2050}(0.15) + \frac{600}{2050}(0.30) \approx 0.194$$

$$\frac{1450}{2050}(0.10) + \frac{600}{2050}(0.20) \approx 0.129$$

Application to the COVID-27 example

$$\mathbb{E}[Y|\text{do}(T=t)] = \mathbb{E}_C \mathbb{E}[Y|t, C] = \sum_c \mathbb{E}[Y|t, c] P(c)$$

Causal Graph



Condition

Treatment	Mild	Severe	Total	Causal
A	15% (210/1400)	30% (30/100)	**16%** (240/1500)	19.4%
B	**10%** (5/50)	**20%** (100/500)	19% (105/550)	**12.9%**
	$\mathbb{E}[Y|t, C=0]$	$\mathbb{E}[Y|t, C=1]$	$\mathbb{E}[Y|t]$	$\mathbb{E}[Y|\text{do}(t)]$

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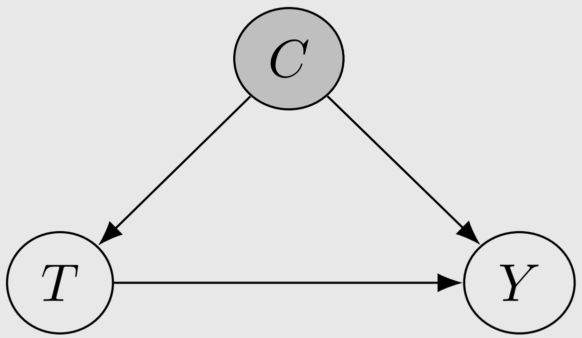

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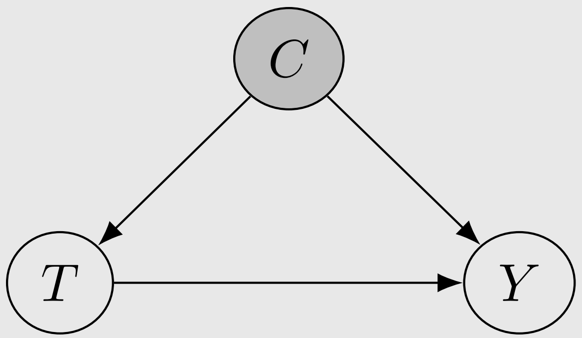

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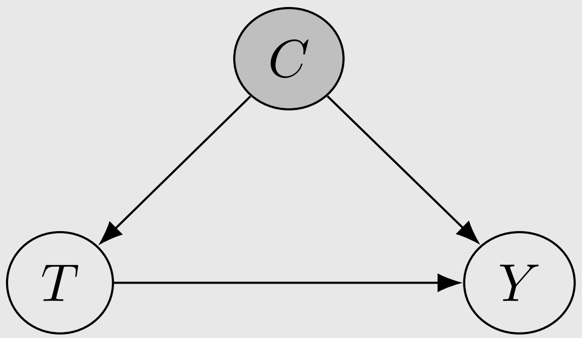

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Application to the COVID-27 example

Condition

Treatment

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Naive (from the Total column):

$$\frac{1400}{1500}(0.15) + \frac{100}{1500}(0.30) = 0.16$$

$$\frac{50}{550}(0.10) + \frac{500}{550}(0.20) = 0.19$$

Causal:

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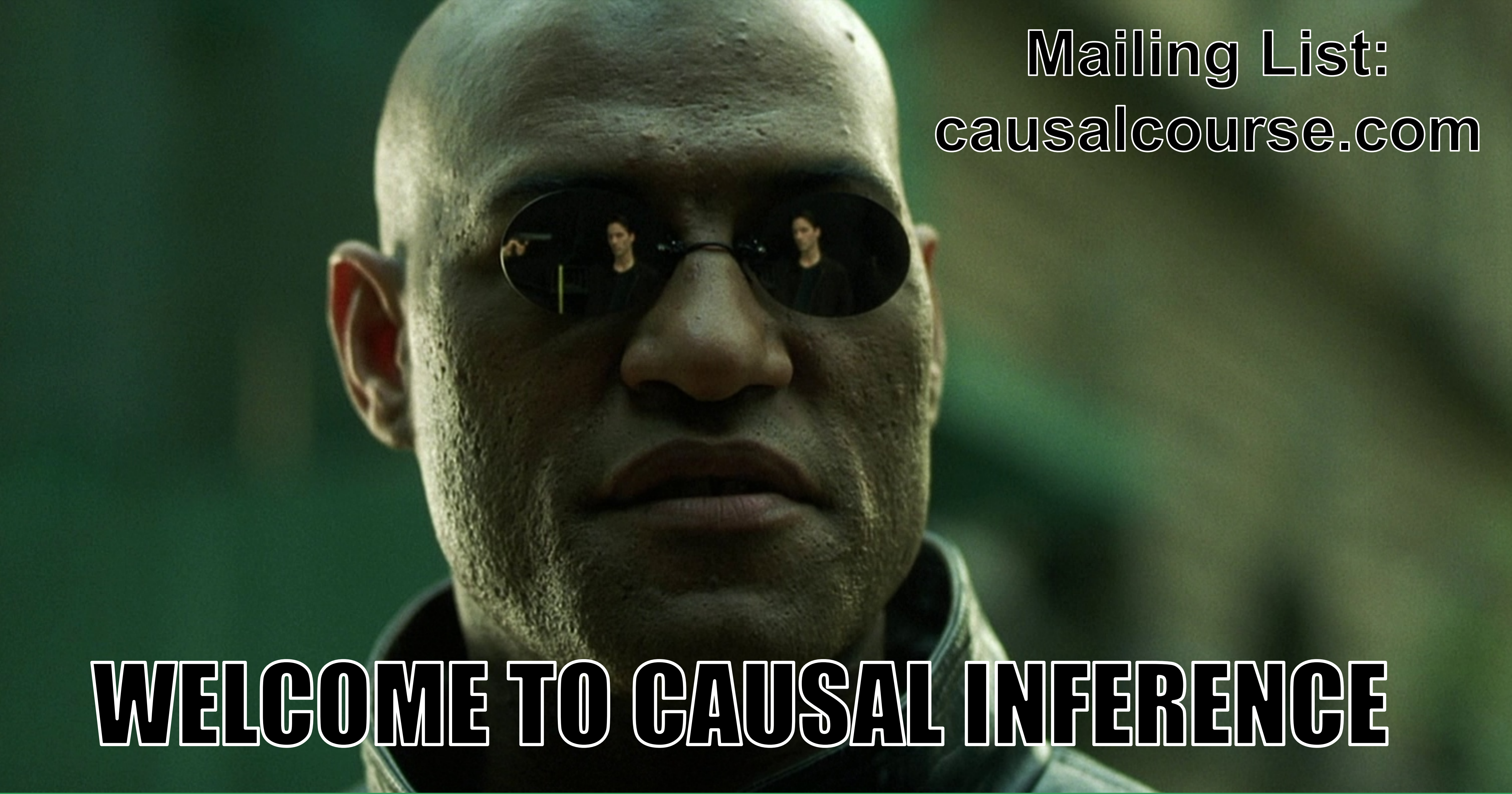
Mailing List:
causalcourse.com
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